Implications of Habit Formation and Preference Change for Free Trade Patterns and Global Economic Growth

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ABSTRACT

This paper constructs a neoclassical global economic growth model with any (finite) number of countries. The model is concerned with endogenous capital accumulation and dynamics of habits and preferences. The production side is based on the neoclassical growth theory, while the consumer behavior is modeled by Zhang’s approach. The habit formation and preference change are strongly influenced by the literature of Ramsey-type growth models with endogenous habits or/and time preference. The dynamics of \( J \)-country global economy is described by \( 2J \) nonlinear autonomic differential equations. We simulate the model for a three-country world economy. The global economy has a unique stable equilibrium for the chosen values of the parameters. The comparative analysis provides some insights into dynamics of economic globalization.

JEL Classification: F11, F21, O41

Keywords: Trade Pattern; Capital Accumulation; Habit Formation; Preference Change.

INTRODUCTION

The contemporary economy is characterized of fast economic globalization, increasingly free mobility of goods and services, and rapid economic developments in different parts of the world. Nevertheless, there are a few economic theories which deal with international economic growth on microeconomic foundation. The purpose of this study is to develop a neoclassical global economic growth model with any (finite) number of countries. The model treats not only endogenous capital accumulation, but also habits and preferences as endogenous variables. The production side is based on the...
neoclassical growth theory, while the consumer behavior is modeled by Zhang’s approach. The habit formation and preference change are strongly influenced by the literature of Ramsey-type growth models with endogenous habits or/and time preference.

A main contribution of this study is to introduce habit formation and preference change in dynamic international trade theory. Many empirical studies demonstrate that incorporating habit formation and preference change in modeling household behavior is statistically significant in explaining consumers’ behavior (e.g., Ferson and Constantinides, 1991; Braun et. al., 1993; Naik and Moore, 1996; Cardi and Müller, 2005). These studies demonstrate that different people have different preferences and preferences are changeable. There are many factors which may attribute to these preference changes. In The Theory of Moral Sentiments, Adam Smith illustrates: “The man who lives within his income is naturally contented with his situation, which, by continual, though small accumulations, is growing better and better every day. He is enabled gradually to relax, both in the rigour of his parsimony and in the severity of his application; and he feels with double satisfaction this gradual increase of ease and enjoyment, from having felt before the hardship which attended the want of them.” Fisher (1930: 72) emphasizes the influence of wealth and income on preference difference: “Poverty bears down heavily on all portions of a man’s expected life. But it increases the want for immediate income even more than it increases the want for future income.” In the literature of economic growth and development, preference change is modeled in the name of time preference in the Ramsey-type growth models. The formal modeling of time preference change was started by Uzawa (1968), Lucas and Stokey (1984) and Epstein (1987) also make time preference change dependent on consumptions. In Becker and Barro (1988), a parent’s generational discount rate is related to their fertility. There are many other studies on the implications of endogenous time preference for economic growth (e.g., Epstein and Hynes, 1983; Obstfeld, 1990; Shin and Epstein, 1993; Palivos et al., 1997; Druegon, 1996, 2000; Stern, 2006; and Dioikitopoulos and Kalyvitis, 2010). The idea of analyzing change in impatience in our study is influenced by this literature. We deviate from the mainstream by taking account of endogenous changes in impatience in an alternative utility proposed by Zhang (1994, 2010).

Another important approach in explaining consumers’ behavior in the literature of economic growth is the habit formation or habit persistence model. The idea was formally proposed by Duesenberry (1949). Becker (1992) accounts the influence of habit on consumers’ behavior as follows: “the habit acquired as a child or young adult generally continue to influence behavior even when the environment changes radically. For instance, Indian adults who migrate to the United States often eat the same type of cuisine they had in India, and continue to wear the same type clothing.” Habit formation is applied to different fields of economics (e.g., Mehra and Prescott, 1985; Sundaresan, 1989; Constantinides, 1990; Croix, 1996; Campbell and Cochrane, 1999; Boldrin et al., 2001, Christiano et al., 2005; Ravn et al., 2006; Huang, 2012). In the literature, ‘catching up with the Joneses’ is often used exchangeable with external habit formation. Economists have recognized the importance of habit formation for explaining trade patterns. An early formal modeling of implications of habit formation on international trade was proposed by Mayer (1980), who built a small-open trade model in discrete time on the basis of the habit formation proposed by Pollak (1970, 1976). Mayer applies the habit formation to explain the enhancement of import elasticities over time. Mayer (1980: 367)
observed: “Consumer habit formation is one possible explanation for countries’ unwillingness to be very import dependent, as well as for their import elasticities to be larger in the long than short run. … This consumption-oriented cause adds a counterpart to the frequently mentioned production adjustment cost as explanation for observed differences between short- and long-run elasticities.” In another application of the Uzawa-type habit formation in a Ramsey growth model, Ikeda and Gombi (1998) analyze the current account dynamics for a small economy. They conclude that if habit persistence is very strong, then the welfare dynamics are sluggish and a harmful hangover effect on welfare may be resulted from a beneficial tax-financed fiscal policy. In a recent article, Gonzalez-Hernandez and Karayalcin (2011) incorporate habit formation and endogenous time preference into an open economic growth model with two economies. There are some other studies on open-economy implications of habit formation (Obstfeld, 1992; Mansoorian, 1993; Gruber, 2004; Kano, 2009). Nevertheless, almost all the formal models on implications of habit formation and preference change for international trade are limited to small open economies with some important variables fixed in global markets. This paper builds a neoclassical economic growth model with endogenous capital accumulation, habit formation and preference change. The rest of the paper is organized as follows. Section 2 defines the multi-country model with capital accumulation, habit formation and preference change. Section 3 shows that the dynamics of the world economy with $J$ countries are given by $2J$ nonlinear autonomous differential equations and simulates the motion of the 3-country world economy. Section 4 conducts comparative dynamic analysis with regard to some parameters. Section 5 concludes the study.

The Neoclassical Multi-country Growth Model

We now build a dynamic one-commodity and multiple-country trade model to examine interdependence between trades and global growth with habit formation and preference change. The model is based on Zhang (1994), except the parts related to habit formation and preference. The global economy is modeled within the framework of a simple international macroeconomic growth model with perfect capital mobility. It is well known that Oniki and Uzawa (1965) proposed a neoclassical growth trade theory with capital accumulation. The model has been used as the core model when economists build neoclassical trade models with endogenous capital accumulation (e.g., Deardorff, 1973, Ruffin, 1979, Findlay, 1984, Frenkel and Razin, 1987, Eaton, 1987, Brecher, et al., 2002, Nishimura and Shimomura, 2002, Sorger, 2003). Our model is specially based on the neoclassical growth trade model with capital accumulation by Ikeda and Ono (1992). We consider that the global economy consists of $J$ countries, indexed by $j = 1, ..., J$. Each country has a constant population, $N_j$, $j = 1, ..., J$. Each country has one sector, producing for consumption and investment. All the countries produce the same single commodity. Households own assets and spend their disposable incomes on consuming and saving. Production sectors employ capital and labor. All markets are perfectly competitive. Factor markets work well. The factors are always in full employment. Saving is undertaken only by households, which implies that all earnings of firms are distributed in the form of payments to factors of production. We neglect the possibility that households hoard output in
the form of non-productive inventories. The commodity is traded without any barriers such as transport costs or tariffs. There is no international migration. The price of the commodity is chosen to be unity and all prices are measured in terms of the commodity. We represent the wage and interest rates in country \( j \) by \( w_j(t) \) and \( r_j(t) \), respectively. In the free trade system, the interest rate is assumed identical throughout the world economy, i.e., \( r(t) = r_j(t) \).

**The production sector**

There are two input factors in each country, capital and labor at each point in time. The production functions are

\[
F_j(t) = A_j K_j^\alpha(t) N_j^\beta, \quad \alpha_j, \beta_j \geq 0, \quad \alpha_j + \beta_j = 1, \quad j = 1, \ldots, J, \tag{1}
\]

where \( A_j \) is the total productivity of the production sector \( \alpha_j \) and \( \beta_j \) are the output elasticities of capital and labor, respectively, in country \( j \). Labor and capital are paid by their marginal products in free competitive markets. The marginal conditions are

\[
r(t) = \frac{\alpha_j F_j(t)}{K_j(t)} - \delta_k, \quad w_j(t) = \frac{\beta_j F_j(t)}{N_j}, \tag{2}
\]

where \( \delta_k \) is the depreciation rate of physical capital in country \( j \).

**The households**

This study uses the approach to consumers’ behavior proposed by Zhang (1994). Let \( \tilde{K}_j(t) \) stand for per capita wealth in country \( j \). The representative household in country \( j \) obtains from the interest payment, \( r \tilde{K}_j \), and the wage payment, \( w_j \). The current income is

\[
y_j(t) = r(t)\tilde{K}_j(t) + w_j(t), \quad j = 1, \ldots, J, \tag{3}
\]

The disposable income is the sum of the current income plus the market value of the household’s wealth, i.e.

\[
\hat{y}_j(t) = y_j(t) + \tilde{K}_j(t). \tag{4}
\]

The representative household uses the disposable income for saving \( s_j(t) \) and consuming \( c_j(t) \). The budget constraint is

\[
c_j(t) + s_j(t) = \hat{y}_j(t). \tag{5}
\]

The representative household’s utility in country \( j, U_j(t) \) is specified as follows

\[
U_j(t) = \theta_j c_j^{\xi_{oj}(t)}(t) s_j^{\lambda_{oj}(t)}(t), \quad \xi_{oj}(t), \quad \lambda_{oj}(t) > 0, \tag{6}
\]

in which \( \xi_{oj}(t) \) and \( \lambda_{oj}(t) \) are respectively the representative household’s utility elasticities of commodity and savings in country \( j \). We call \( \xi_{oj}(t) \) and \( \lambda_{oj}(t) \) respectively propensities to consume the commodity and to save. In (6), \( \theta_j \) is a positive parameter. Maximizing \( U_j(t) \) subject to (5) yields
\[ c_j(t) = \xi_j(t) \hat{y}_j(t), \quad s_j(t) = \lambda_j(t) \hat{y}_j(t), \]  

(7)

in which

\[ \xi_j(t) = \rho_j(t) \xi_o(t), \quad \lambda_j(t) = \rho_j(t) \lambda_o(t), \quad \rho_j(t) = \frac{1}{\xi_o(t) + \lambda_o(t)}. \]

We call \( \xi_j(t) \) and \( \lambda_j(t) \) respectively the relative propensities to consume and to save. It can be seen that it is not the propensities, but the relative propensities that matter in allocating the disposable income. In our approach we do not assume identical preference of the households across the countries. In the traditional neoclassical growth trade models, it is often required that countries have similar preferences. For instance, in a trade model Bianconi and Turnovsky (1997: 61) give the following reason why it is necessary to consider that different countries have similar differences: "We shall assume that the rate of time discount of the foreign resident equals that of the domestic agent. As is well known, under the assumption of perfect foresight and perfect capital markets, this assumption is necessary in order for a well-defined steady state to exist." This kind of requirements on similarity in preferences of different countries is obviously not proper for explaining different behavior of households.

**Wealth accumulation**

According to the definitions of \( s_j(t) \) the wealth accumulation of the representative household in country \( j \) is given by

\[ \tilde{K}_j(t) = s_j(t) - \bar{K}_j(t). \]

(8)

**Full employment of production factors**

The total capital stocks employed by all the sectors in the global economy \( K(t) \) is equal to the world wealth

\[ K(t) = \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \bar{K}_j(t) = \sum_{j=1}^{J} \bar{K}_j(t) N_j, \]

(9)

where \( K_j(t) \) is the capital stock employed by country \( j \) and \( \bar{K}_j(t) \) is the capital stock owned by country \( J \). The total wealth of the national economy is the sum of the wealth owned by all the households. We introduce \( B_j(t) \) as the value of country \( j \)'s net foreign assets at \( t \). The income from the net foreign assets, \( E_j(t) \) which may be either positive, zero, or negative, is equal to \( r(t)B_j(t) \). According to the definitions of the national wealth, the capital stocks employed by the economy and the net foreign assets, we have

\[ \bar{K}_j(t) = K_j(t) + B_j(t). \]

(10)
A country’s current balance at time $t$ is the change in the value of its net claims over the rest of the world – the change in its net foreign assets. If $\hat{B}_j(t) > 0$ the economy is lending (in this case we say that the current account balance is in surplus); if $\hat{B}_j(t) < 0$ the economy is borrowing (the current account balance is in deficit); and if $\hat{B}_j(t) = 0$ the economy is neither borrowing nor lending (the current account balance is in balance). We introduce variables to measure trade balances $E_j(t) \equiv (K_j(t) - K_j(t))r(t)$.

If $E_j(t) > (=, <) 0$, we say that country $j$’s trade is in surplus (balance, deficit).

**The time preference and the propensity to hold wealth**

We model preference change by introducing endogenous changes in the propensity to own wealth and propensity to consume. There are many formal models about preference change. But as far as growth theory is concerned, the mainstream of modeling endogenous preference is to introduce proper dynamic mechanisms for change in the time preference in Ramsey-type models. Our modeling of the propensity to save is strongly influence by the traditional approach. Before modeling the propensity to save in Zhang’s approach, we introduce the idea of time preference in the traditional literature. In Chang *et al.* (2011), the discounted lifetime utility is given by

$$\int_0^\infty u(c, m)e^{-\rho(t)} dt,$$

where $u$ is the utility function, $c$ is consumption, and $m$ is holdings of real money balances. The preference change is reflected in change time preference $\rho(t)$ (see also, Uzawa, 1968; Epstein, 1987; Obstfeld, 1990; and Shi and Epstein, 1993). Chang *et al.* specify the cumulated subjective discount rate as follows

$$\rho(t) = \int_0^t \Delta(u(s))ds,$$

where $\Delta > 0$ is an instantaneous subjective discount rate at time $s$ which satisfies $\Delta' > 0$, $\Delta'' > 0$ and $\Delta - u\Delta' > 0$. Taking derivatives of the above equation with respect to time yields

$$\dot{\rho}(t) = \Delta(u(t)).$$

It should be noted that as far as formal modeling is concerned, this way of endogenous time preference was proposed by Uzawa (1968). Nevertheless, Uzawa’s preference change implies that richer people are more impatient. Some economists criticize Uzawa’s specified form of the preference change as being “arbitrary and even counterintuitive” (Persson and Svensson, 1985: 45; see also, Blanchard and Fischer, 1989; Orphanides and Solow, 1990; Das, 2003; and Hirose and Ikeda, 2008). Although the study does not directly follow the Ramsey approach, Zhang (2012) develops his preference change on the basis of the ideas about time preference within the Ramsey framework. Following Zhang, this study proposes the dynamics of the propensity to save as follows
\[
\overline{\lambda_0}(t) = \lambda_j + \lambda_{wj}(t) + \lambda_{kj}k_j(t)
\]

(11)

where \( \overline{\lambda_0} \) is a parameter. When \( \lambda_j = \lambda_{wj} = 0 \), the propensity to hold wealth is constant. In this case, the model is similar to the two-country model by Zhang (1994). If we follow the assumption that the rate of time preference is positively related to wealth, for instance, accepted by Smithin (2004) and Kam and Mohsin (2006), then \( \lambda_{wj} = 0 \) and \( \lambda_{kj} \).

The habit formation and the propensity to consume

As mentioned in the introduction, empirical studies show that habits strongly affect people’s consumption behavior. As consumption patterns affect economic growth and development, it is reasonable to expect that there are studies incorporating habit and its dynamics in the Ramsey-type growth model – the core growth model in the neoclassical economic growth theory (see, for instance, Deaton and Muellbauer, 1980; Carroll et al. 1997; Fuhrer, 2000; Carroll, 2000; Kozicki and Tinsley, 2002; Amano and Laubach, 2004, Corrado and Holly, 2011). To explain how economists incorporate habits in the growth theory, we introduce a habit formation in continuous time in the Ramsey-type growth models (e.g., Alvarez-Cuadrado et al., 2004; and Gómez, 2008)

\[
h(t) = \rho \int_{-\infty}^{t} e^{\theta(s-t)} C^\phi(s) \overline{C}^{1-\phi}(s) ds, \quad \rho > 0, \quad 0 \leq \phi \leq 1,
\]

where \( C(t) \) is the consumer’s consumption and \( \overline{C}(t) \) is the economy-wide average consumption. A larger value for \( \theta \) would involve lower weights given to more distant values of the levels of consumption. The relative weights of consumption at different times are reflected in the value of the parameter. Taking derivatives of the above equation with respect to time, we have

\[
\dot{h}(t) = \dot{h}_0 \left[ C^\phi(s) \overline{C}^{1-\phi}(s) - h(t) \right].
\]

(12)

In this approach, the case \( \theta = 0 \) corresponds to the model with external habits. If \( \theta = 1 \) the habit formation is called the internal habits. If \( 0 < \theta < 1 \) habit formation is due to both the consumer’s and average past consumption. Following Zhang (2012), this study considers that habit affects the propensity to consume. Applying the concept of habit stock, we model how the past consumption affects the current preference. This study applies Zhang’s following habit formation

\[
\dot{h}_j(t) = \dot{h}_j(0) \left[ c_j(t) - h_j(t) \right].
\]

(13)

Equation (12) corresponds to the habit formation in the traditional Ramsey-type growth model with internal habits. If the current consumption is higher than the level of the habit stock, then the level of habit stock tends to rise, and vice versa. It should be noted that like (12), we may construct a more general form of habit formation

\[
\dot{h}_j(t) = \dot{h}_j(0) \left[ c_j^\varphi(t) \overline{C}^{1-\varphi}(t) - h_j(t) \right], \quad 0 \leq \varphi \leq 1,
\]
where
\[
\bar{c}(t) = \text{Max}_j \{c_j(t)\}, \text{ or } \bar{c}(t) = \frac{1}{J} \sum_{j=1}^{J} c_j(t).
\]

For simplicity of analysis, this study is limited to (13). The propensity to consume is assumed a function of the habit stock in the following way
\[
\xi_{0j}(t) = \bar{\xi}_j + \xi_{wj} w(t) + \xi_{hj} h_j(t),
\]
where \(\bar{\xi}_j > 0, \xi_{wj} \text{ and } \xi_{hj} \geq 0\) are parameters. If \(\xi_{wj} = 0\) and \(\xi_{hj} = 0\) the propensity is constant. The term \(\xi_{wj} w_j(t)\) implies that the propensity to consume is affected by the wage. If \(\xi > (\prec)\) 0, then a rise in the wage rate raises (reduces) the propensity to consume. It is reasonable to assume \(\xi_{wj} \geq 0\) The term \(\xi_{hj} h_j(t)\) implies that a rise in the habit stock causes the propensity to consume to rise, and vice versa.

We thus built the multi-country growth model with habit formation and preference change. We now examine the properties of the system.

**Global Economic Dynamics**

The appendix confirms that the motion of the economic system with \(J\) national economies is described by 2\(J\) autonomous differential equations with 2\(J\) variables, \(K_i(t), \{\bar{K}_j(t)\}_j\), and where
\[
\{\bar{k}_j(t)\} = [\bar{k}_2(t), ..., \bar{k}_J(t)]
\]

Once we obtain the solution of the time-invariant system, we can determine all the other variables in the global economy at any point in time. We prove the following lemma in the appendix.

**Lemma**

The motion of the global economic system is given by the following 2\(J\) autonomous differential equations with \(K_i(t), \{\bar{K}_j(t)\}_j\), and \([h_j(t)]_J\) as the variables
\[
\begin{align*}
\dot{K}_1 &= \Phi_1(K_1, \{\bar{K}_j\}_j, h_1), \\
\dot{\bar{k}}_j &= \Phi_j(K_1, \{\bar{K}_j\}_j, h_j), \quad j = 2, ..., J, \\
\dot{h}_j &= \Lambda_j(K_1, \{\bar{K}_j\}_j, h_j), \quad j = 1, ..., J,
\end{align*}
\]
where \(\Phi_j\) and \(\Lambda_j\) are functions of \(K_i(t), \{\bar{K}_j(t)\}_j\), and \([h_j(t)]_J\) defined in the appendix. The following procedure gives all the other variables as functions of \(K_i(t), \{\bar{K}_j(t)\}_j\), and \([h_j(t)]_J\) : \(r(t)\) and \(w_j(t)\) by (A3) \(\rightarrow k_j\) by (A2) \(\rightarrow \xi_{wj}\) by (A4) \(\rightarrow k_i(t)\) by (A5) \(\rightarrow \lambda_{oj}(t)\) by (A6) \(\rightarrow \lambda_j(t)\) and \(\xi_j(t)\) by (A7) \(\rightarrow \dot{y}_j(t)\) by (A8) \(\rightarrow c_j(t)\) and \(s_j(t)\) by (A9) \(\rightarrow F_j(t)\) by (1).
From the procedure in the lemma we can get the value of any variable at any point in
time as functions of \( f(\tau), \{ \bar{\kappa}_i(\tau) \}, \) and \( \{ h_i(t) \} \)\textsuperscript{[7]}\textsuperscript{\textsuperscript{[7]}}. The autonomous differential equations are
nonlinear. It is not easy to get analytical solution of the time-invariant system. Nevertheless,
we can use computer to plot the paths of the system over time. To simulate the model, we
choose the following parameter values for a three-country global economy

\[
\alpha_{ij} = 0.3, \lambda_{ij} = 0.1, \lambda_{k} = 0.02, h_j = 0.1, \xi_{ij} = 0.01, \xi_{hj} = 0.04
\]

\[
\begin{array}{c}
N_1 = 2, A_1 = 1.3, \bar{x}_1 = 0.7, \bar{z}_1 = 0.09, \delta_{x1} = 0.04 \\
N_2 = 3, A_2 = 0.9, \bar{x}_2 = 0.62, \bar{z}_2 = 0.1, \delta_{x2} = 0.05 \\
N_3 = 6, A_3 = 0.7, \bar{x}_3 = 0.65, \bar{z}_3 = 0.12, \delta_{x3} = 0.05
\end{array}
\]

Country 1 has the highest level of the total productivity. Country 2’s level of the total
productivity is lower than the level of the productivity in country 1 and higher than in country
3. We consider that country 1 is technologically more advanced than country 2 and country 2
more advanced than country 3. Country 1 has the smallest population and country 3 has the
largest population. In our neoclassical model the population size has no impact on the per-
capita variables, even though it affects the aggregate variable levels. The chosen values of the
population sizes will not affect our main conclusions. The output elasticities of capital and labor
are the same among the countries. Although the specified values are not based on empirical
observations, the choice does not seem to be unrealistic. For instance, some empirical studies
on the US economy estimate the output elasticity of capital in the Cobb-Douglas production
approximately equal to \( \) (for instance, Miles and Scott, 2005; and Abel \textit{et al}. 2007). Although
the chosen values of the preference parameters are not empirically based, we choose the
coefficients associated with the wage and wealth very small so that we may effectively analyze
the effects of changes in these coefficients on the global economic growth. To simulate the
motion of the system, we fix the following initial conditions:

\[
K_1(0) = 3, \bar{k}_2(0) = 5, \bar{k}_3(0) = 3, h_1(0) = 2, h_2(0) = 1.1, h_3(0) = 0.8.
\]

Figure 1 plots the simulation result. We see that all the three countries’ habit stocks of
consumption fall, even though country 1’s consumption level lowers, while the other two
countries’ consumption levels rise. Country 1’s propensity to consume initially rises and
subsequently falls, while the other two countries’ propensities to consume rise slightly. Country
1’s propensity to save falls, while the other two countries’ propensities to save rise slightly. It
should be noted that the change patterns in the relative propensities are different from these
of the propensities. For instance, the change in the propensity to save is monotonic, while the
relative propensity to save is of waveform. This occurs because the relative strengths affecting a
country’s two propensities change over time. As the propensity to save lowers in the long term
(to compare respectively with their initial conditions) and the propensity to consume becomes
higher, each country’s wealth is lessened. Consequently the global wealth and output are
diminished. The capital stock and output level of each country are lowered. Although country
1’s per capita wealth falls, the per capita levels in the other two countries are augmented. This
occurs as country 1’s relative propensity falls more than the other two countries’. As the global
wealth is lessened, the rate of interest is increased. The wage rates are slightly affected. Country 1’s trade balance is deteriorated, while the other two countries’ trade balances are improved.

From Figure 1, we see that all the variables tend to become stationary in the long term. In fact, further simulation with longer time than shown in Figure 1 confirms that the global economy approaches an equilibrium point. The equilibrium values of the variables are listed below

\[
\begin{align*}
F &= 15.93, \quad K = 59.08, \quad r = 0.035, \\
F_1 &= \begin{pmatrix} 5.29 \\ 4.44 \\ 6.20 \end{pmatrix}, \quad K_1 = \begin{pmatrix} 21.28 \\ 15.77 \\ 22.03 \end{pmatrix}, \\
E_1 &= \begin{pmatrix} -0.06 \\ 0.03 \\ 0.03 \end{pmatrix}, \quad \lambda_1 = \begin{pmatrix} 0.817 \\ 0.819 \\ 0.817 \end{pmatrix}, \\
\xi_1 &= \begin{pmatrix} 0.183 \\ 0.181 \\ 0.184 \end{pmatrix}, \\
\lambda_{01} &= 0.88, \quad \xi_{01} = 0.20, \\
\lambda_{02} &= 0.72, \quad \xi_{02} = 0.16, \\
\lambda_{03} &= 0.72, \quad \xi_{03} = 0.16, \quad \gamma_1 = 1.85, \quad \gamma_2 = 1.04, \quad \gamma_3 = 0.72, \quad \gamma_4 = \begin{pmatrix} 2.19 & 1.23 & 0.88 & 9.79 \end{pmatrix}, \\
\gamma_5 &= \begin{pmatrix} 5.55 \\ 3.81 \end{pmatrix}.
\end{align*}
\]

It is straightforward to calculate the following six eigenvalues

\[-0.1 \pm 0.5i, -0.10 \pm 0.06i, -0.09 \pm 0.07i\]

As the six eigenvalues have real negative parts, the equilibrium point is locally stable. Hence, the system always approaches its equilibrium point if it is not far from the equilibrium point. This property is significant as it ensures the validity of comparative dynamic analysis.

**Comparative Dynamic Analysis**

From the analysis in the previous section we are ensured that the economic system has a unique locally stable equilibrium. This means that we can effectively conduct comparative dynamic
This section shows how the paths of the global economy are affected by changes in some parameters. By the way, we remark that as the system contains many variables which nonlinearly interact with each other in a very complicated manner over time, it is not easy to accurately interpret how all these variables interact over time.

**Country 1’s household giving lower weights to more distant values of consumption**

We first study the case where country 1’s household gives lower weights to more distant values of the levels of consumption as follows: $h_{01} : 0.1 \Rightarrow 0.3$ This also implies that the habit stock converges to the current level of consumption faster. Figure 2 plots the simulation results. In the figure the symbol $\Delta x(t)$ represents the change rate of the variable, $x(t)$ in percentage due to changes in some parameter value. As demonstrated in the figure, the change in the weight distribution has no impact on the equilibrium of the global economy, even though the transitional paths of the variables vary as the adjustment speed is changed. As the household in country 1 lowers the weights of more distant values of consumption, initially country 1’s habit stock deviates from the original path as plotted in Figure 1 and moves upwards. The rise in the habit stock draws the country’s consumption level up. This change is association the falling in the propensity to save and the rising in the relative propensity to consume in country 1. As country 1’s consumption level is enhanced, the country saves less and world’s capital is reduced. As the global capital is lessened, the rate of interest is enhanced and the output levels of the three national economies are reduced. The propensities and relative propensities of the other two countries are slightly affected by the parameter change. The wage rates of the three national economies are initially. Country 1’s trade balance is slightly deteriorated, while the other two countries’ trade balances are improved. Nevertheless, the subsequent development reverses the change directions of these variables. As the output levels and global wealth are lessened, country 1’s habit stock begins to fall, which is associated with falling in the country’s consumption. As the consumption level lowers, more savings are made and the global wealth starts to increase. The capital stocks employed by and the output levels of the three economies all begin to increase. The variables oscillate around the original paths before they come back to their original equilibrium values.

![Figure 2 Country 1’s Household Giving Lower Weights to More Distant Values of Consumption](image-url)
Country 1’s wealth effect on the propensity to save becoming stronger

We now simulate what will happen in the global economy if country 1’s wealth effect on the propensity becomes stronger as follows: $\lambda_{1k} : 0.02 \Rightarrow 0.02$. The initial effect of the exogenous disturbance enhances country 1’s propensity and relative propensity to save and lowers the relative propensity to consume. The global output and wealth are augmented. As capital input is available in the global market with lower cost, all the three economies employ more capital inputs, increase their wage rates, and enhance the output levels. Countries 2's and 3's propensities are slightly affected. Consequently, country 1’s per capita is increased and the other two countries’ per capita and consumption levels are only slightly affected. As its propensity to consume falls initially, country 1’s consumption level lowers, which is associated with lessening in the habit stock. But subsequently the both consumption level and habit stock of consumption are enhanced in association with rising in per capita wealth. Different from the monotonically increasing pattern in the consumption level after its initial falling, country 1's habit stock exhibits a waveform. Country 1's trade balance is improved, while the other two countries’ trade balances are deteriorated.

![Figure 3. Country 1’s Wealth Effect on the Propensity to Save Becoming Stronger](image)

Country 1’s propensity to consume being more strongly affected by habits

We now investigate the impact of the following change in how country 1's propensity to consume is influenced by habits of consumption: $\zeta_{h1} : 0.04 \Rightarrow 0.05$. The relative propensity to consume is enhanced and the relative propensity to save is lowered. The propensities and relative propensities of the other two countries are slightly affected. As country 1 saves less from its disposable income, the wealth accumulated by the country falls, even though the other two countries’ wealth levels are slightly changed by country 1's preference change. The rate of interest rises and the wage rates fall. The global wealth and output level falls. The capital inputs and output levels of each national economy are lessened. Country 1’s trade balance
is slightly deteriorated and the other two countries’ trade balances are slightly improved. Although country 1’s consumption and habit stock rise initially, subsequently they fall in association with falling in the per capita wealth. It should be noted that although their total output levels are reduced, country 2’s and country 3’s per capita wealth and consumption levels are slightly affected. This occurs partly because the rate of interest is increased. Hence, country 1’s preference change lowers its own living standards, but has little impacts on the other countries’ wealth and consumption.

Figure 4. Country 1’s Propensity Being More Strongly Affected by Habits

The wage rates having positive effects on the propensities to save in the three countries

We now examine what happen to the global economy if the three countries simultaneously change their preference in following way: $\lambda_{w1}, \lambda_{w2}, \lambda_{w3} : -0.01 \rightarrow 0.01$. As all the economies increase their propensities to save rather than reduce them as their wage rates are increased, it is reasonable to expect that the global economy accumulates more wealth. Each country’s relative propensity to save rises and relative propensity to consume falls. Country 1’s trade balance is slightly deteriorated and the other two countries’ trade balances are slightly improved. The capital cost falls and the wage rates rise. The per capita wealth in each country is increased. The capital stock employed and output level of each country are augmented. Although the consumption level and habit stock of each economy initially fall, these variables subsequently are all increased.
CONCLUSIONS

This paper built a neoclassical global economic growth model with any (finite) number of countries. The model treats not only endogenous capital accumulation, but also habits and preferences as endogenous variables. This study is focused on habit formation and preference change with capital accumulation. The dynamics of a J-country global economy is described by 2J autonomic differential equations. As it is not easy to find analytical solution of the high dimensional nonlinear time-invariant system, we simulated the model. We identified the existence of a unique equilibrium and showed the local stability with the chosen values of the parameters. We conducted comparative dynamic analysis with regard to some parameters for a three-country world economy. The comparative analysis provided some insights into dynamics of economic globalization. For instance, when country 1’s household gives lower weights to more distant values of the levels of consumption, although the change in the habit formation speed has no impact on the equilibrium of the global economy, the transitional paths of the variables are affected as follows: country 1’s habit stock is augmented initially and the rise in the habit stock draws the country’s consumption level up; the change is associated with falling in the propensity to save and the rising in the relative propensity to consume in country 1; as country 1’s consumption level is enhanced, the country saves less and world’s capital is reduced; the rate of interest is enhanced and the output levels of the three national economies are reduced; the propensities and relative propensities of the other two countries are slightly affected; the wage rates of the three national economies are initially reduced; country 1’s trade balance is slightly deteriorated, while the other two countries’ trade balances are improved; the subsequent development reverses the change directions of these variables. The variables oscillate around the original paths before they come back to their original equilibrium values. As the model is built on the basis of a few approaches in the literature of economic theory and there are many models in each of the approaches, it is straightforward to generalize or extend the model. For instance, it is important to introduce currencies and exchange rates into the model.
Appendix Proving Lemma 1

First inserting (1) in (2) we get

\[ r = \frac{\alpha_j A_j K_j^{\alpha_j} N_j^{\beta_j}}{K_j} - \delta_k = \frac{\alpha_1 A_1 K_1^{\alpha_1} N_1^{\beta_1}}{K_1} - \delta_{k1}, \quad j = 2, ..., J. \]  

(A1)

Solve (A1)

\[ K_j = \phi_j(K_1) = \left[ \left( \frac{\alpha_1 A_1 K_1^{\alpha_1} N_1^{\beta_1}}{K_1} - \delta_{k1} + \delta_k \right) \frac{1}{\alpha_j A_j N_j^{\beta_j}} \right]^{-1/\beta_j}, \quad j = 1, ..., J, \]  

(A2)

where \( \Phi(K_1) = K_1 \). From (A2) and (2), we have

\[ r = \phi_1(K_1) \equiv \frac{\alpha_1 A_1 N_1^{\beta_1}}{K_1^{\beta_1}} - \delta_{k1}, \quad w_j = \tilde{\phi}_j(K_1) \equiv \frac{\beta_j A_j \phi_j^o}{N_j^{\alpha_j}}. \]  

(A3)

From (A3) and (13), we get

\[ \xi_{0_j}(K_1, h_j) = \xi_j + \xi_k w_j + \xi_k h_j. \]  

(A4)

From (9), we get

\[ \bar{k}_1 = \phi_0(K_1, \{\bar{k}_j\}) = n_1 \sum_{j=1}^{J} \phi_j - \sum_{j=2}^{J} n_j \bar{k}_j, \]  

(A5)

where

\[ n_1 \equiv \frac{1}{N_1}, \quad n_j \equiv \frac{N_j}{N_1}, \quad j = 2, ..., J. \]

From (11) and (A5), we have

\[ \lambda_{q_j}(K_1, \{\bar{k}_j\}) = \bar{\lambda}_q + \lambda_{w1} w_1 + \lambda_{k1} \bar{k}_1, \]

\[ \lambda_{0_j}(K_1, \bar{k}_j) = \bar{\lambda}_j + \lambda_k w_j + \lambda_k \bar{k}_j, \quad j = 2, ..., J. \]  

(A6)

From (A6), and the definitions of \( \xi_j \) and \( \lambda_j \) we get

\[ \xi_j(K_1, \{\bar{k}_j\}, h_j) = \rho_j(K_1, \{\bar{k}_j\}, h_j), \]

\[ \lambda_j(K_1, \{\bar{k}_j\}, h_j) = \rho_j(K_1, \{\bar{k}_j\}, h_j), \]  

(A7)

\[ \rho_j(K_1, \{\bar{k}_j\}, h_j) = \frac{1}{\xi_{0_j}(K_1, \{\bar{k}_j\}, h_j) + \lambda_{0_j}(K_1, \{\bar{k}_j\}, h_j)}. \]
From (A1), (A5), and the definitions of $\hat{y}_j$ we solve
\[
\hat{y}_j(K_1, \{\bar{K}_j\}) = (1 + r\bar{K}_j + w_j, \ j = 1, \cdots, J. \quad (A8)
\]

From (A7), (A8), and (7), we have
\[
c_j(K_1, \{\bar{K}_j\}, h_j) = \xi_j(K_1, \{\bar{K}_j\}, h_j)\hat{y}_j(K_1, \{\bar{K}_j\}), \\
s_j(K_1, \{\bar{K}_j\}, h_j) = \lambda_j(K_1, \{\bar{K}_j\}, h_j)\hat{y}_j(K_1, \{\bar{K}_j\}). \quad (A9)
\]

Inserting (A5) and (A9) in (8) and (12), we get
\[
\hat{\kappa}_1 = \Phi_0(K_1, \{\bar{K}_j\}, h_1) = s_1 - \bar{K}_1, \quad (A10)
\]
\[
\hat{\kappa}_j = \Phi_j(K_1, \{\bar{K}_j\}, h_j) = s_j - \bar{K}_j, \ j = 2, \cdots, J, \\
\hat{h}_j = \Lambda_j(K_1, \{\bar{K}_j\}, h_j) = h_{j0}(c_j - h_j), \ j = 1, \cdots, J. \quad (A11)
\]

Taking derivatives of (A5) with respect to time
\[
\hat{\kappa}_1 = \left(\sum_{j=1}^J \frac{d\phi_j}{dK_1}\right)n_1\hat{K}_1 - \sum_{j=2}^J n_j\hat{K}_j, \quad (A12)
\]

Substituting (A11) and (A12) into (A10) yields
\[
\hat{K}_1 = \Phi_1(K_1, \{\bar{K}_j\}, h_1) = N_1\left(\Phi_0 + \sum_{j=2}^J n_j\Phi_j\right)\left(\sum_{j=1}^J \frac{d\phi_j}{dK_1}\right)^{-1}. \quad (A13)
\]

From (A11) and (A13), we thus proved the lemma.

**REFERENCES**


